* H D Z - 0 0 3 - 1 1 6 3 0 0 4 *

**HDZ-003-1163004** **Seat No.** ____________

**M. Sc. (Sem. III) (CBCS) Examination**

**November/December – 2017**

**Mathematics : MATHS. CMT-3004**

*(Disctrete Mathematics) (New Course)*

**Faculty Code : 003**
**Subject Code : 1163004**

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Answer all the quesitons.
(2) Each question carries 14 marks.

**1** Answer any Seven : **7×2=14**

(a) Let $A$ be a nonempty set. Define the concept of the free semigroup generated by $A$.

(b) Let $A = \{0,1\}$. Show that the following expressions are regular expressions over $A$.

(i) $0^* (0 \vee 1)^*$

(ii) $(01)^*(01 \vee 1^*)$

(c) Define a complemented lattice and illustrate it with an example.

(d) Let $f : (S, *) \to (T, *')$ be a homomorphism of semigroups. If $f$ is onto and if $(S, *)$ is a monoid, then show that $(T, *')$ is a monoid.

(e) Define a Boolean Algebra. State the reason why the diamond lattice is not a Boolean Algebra.

(f) Let $L \subseteq \{x, y\}^*$. When is $L$ said to be a type 2 language over $\{x, y\}$ ?

(g) Define a (i) phrase structure grammar and a (ii) Moore machine.

(h) Define a machine congruence on a finite state machine.

(i) State Kleene's theorem.

(j) Define a modular lattice. Illustrate that a finite lattice need not be modular.

**2** Answer any Two : **2×7=14**

(a) State and prove the fundamental theorem of homomorphism of semigroups.

(b) Let $(L, \leq)$ be a lattice. Show that (L, $\leq$) is distributive if and only if for all.

$a, b, c \in L, (a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \wedge c) \wedge (c \vee a)$

(c) Let $n \geq 1$ and let $f : B_n \rightarrow B$. Prove that $f$ is produced by a Boolean expression.

**3** (a) Let $G$ be a group and let $H$ be a normal subgroup of $G$. Let $R$ be a relation defined on $G$ by $aRb$ if and only if $ab^{-1} \in H$. Prove that R is a congruence relation on $G$. **5**

(b) Let $V$ be a vector space over a field $F$. Show that the lattice of subspaces of $V$ is modular. **5**

(c) Let $f : A \rightarrow B$ be a bijection. If $(A, \leq_A)$ is a partially ordered set, then show that we can define a relation $\leq_B$ on $B$ such that $(B, \leq_B)$ is a poset and $f : (A, \leq_A) \rightarrow (B, \leq_B)$ is an isomorphism of posets. **4**

**OR**

**3** (a) Let $n \geq 1$. Prove that $D_n$, the lattice of positive divisors of n is distributive. **5**

(b) Let $G = (V, S, v_0, \mapsto)$ be a phrase structure grammar in which $\{v_0, x, y, z\}$, $S = \{x, y, z\}$, and the productions are given by **5**

(1) $v_0 \mapsto xv_0$,

(2) $v_0 \mapsto yv_0$, and

(3) $v_0 \mapsto z$.

Find $L(G)$

(c) Let $R$ be a symmetric relation defined on a nonempty set $A$. Prove that $R^{\infty}$ is symmetric. **4**

**4** Answer any Two : **2×7=14**

(a) Let $(L, \leq)$ be a finite Boolean Algebra. Prove that the number of atoms of $(L, \leq)$ is equal to the number of coatoms of $(L, \leq)$.

(b) Let $M = (S, I, \mathcal{F}, s_0, T)$ be a Moore machine. Prove that there exists a type 3 phrase structure grammar $G$ with $I$ as its set of terminal symbols such that $L(M) = L(G)$.

(c) Let $M = (S, I, \mathcal{F}, s_0, T)$ be a Moore machine. If $R$ is the w-compatibility relation defined on $S$, then show that $R$ is a machine congruence on $M$ and $L(M) = L(M/R)$.

**5** Answer any Two : **2×7=14**

(a) Let $M = (S, I, \mathcal{F}, s_0, T)$ be a Moore machine. If $w \in L(M)$ is such that $l(w) \geq |S|$, then show that there exist $w_1, w_2, w_3 \in I^*$ such that $l(w_2) > 0, w = w_1 w_2 w_3$ and $w_1 w_2^k w_3 \in L(M)$ for all $k \geq 0$.

(b) For the languages given in

(i) and (ii) below, construct a phrase structure grammar $G$ such that $L(G) = L$.

(i) $L = \{a^n b^m \mid n \geq 1, m \geq 3\}$

(ii) $L = \{x^n y^m \mid n \geq 2, m \geq 0 \text{ and even}\}$

(c) Let $(L, *)$ be a commutative semigroup in which $a * a = a$ for all $a \in L$. Prove that the relation $\leq$ defined on $L$ by $a \leq b$ if and only if $a * b = b$ is a partial order and for any $a, b \in L, a * b$ is the least upper bound of $\{a, b\}$ in $(L, \leq)$.

(d) Let $M = (S, I, \mathcal{F}, s_0, T)$ be a Moore machine. For each $n \geq 0$, let $R_n$ be the relation defined on $S$ by $s_i R_n s_j$ if and only if $s_i$ and $s_j$ are w-compatible for all $w \in I^*$ with $l(w) \leq n$. Let $k \geq 0$ and let $s, t \in S$. Show that the following statements are equivalent :

(i) $sR_{k+1}t$

(ii) $sR_k t$ and $f_x(s) R_k f_x(t)$ for each $x \in I$.